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A SEMI-ANALYTICAL METHOD FOR THE ANALYSIS OF THE INTERACTIVE BUCKLING OF THIN-WALLED ELASTIC STRUCTURES IN THE SECOND ORDER APPROXIMATION

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Abstract—The investigation is concerned with a semi-analytical method for the interactive buckling analysis of thin-walled closed and open cross-section structures in the second order approximation based on the linear analysis. An improved "lower bound" approach by Koiter and Pignataro [Koiter, W. T. and Pignataro, M. (1976). A general theory for the interaction between local and overall buckling of stiffened panels. *WTHD Report No. 556*, Delft University, p. 49] enables us to determine the overall flexural stiffness of a beam-column after its local buckling. The theoretical basis of the present approach is discussed and some typical examples are considered. Copyright © 1996 Elsevier Science Ltd

NOTATION

a_{ijJ}, b_{ijkJ}	three and four-index coefficients in the nonlinear equilibrium equations by eqn (5) (Byskov and Hutchinson (1977))
$\tilde{b}_{KKKK}$	eqn (7)
b_i	width of the i-th wall of the column
h_i	thickness of the i-th wall of the column
l	length of the column
m	number of axial half-waves of mode n
n	number of mode
N	number of interacting modes
$\bar{N}$	force field
N_{ix}, N_{iy}, N_{ixy}	in-plane stress resultants for the i-th wall
N_{ix}^{o}	pre-buckling in plane stress for the i-th wall
$\bar{U}$	displacement field
u_i, v_i, w_i	displacement components of middle surface of the i-th wall
$u_i^{(n)}$, $v_i^{(n)}$, $w_i^{(n)}$	buckling displacement fields
η_K	coefficient of the reduced flexural rigidity
λ	scalar load parameter
λ_n	value of λ at bifurcation mode number n
λ_s	maximum value of λ for imperfect column
ν	Poisson's ratio ($\nu = 0.3$)
ξ_n	amplitude buckling mode number n
$\bar{\xi}_n$	imperfection amplitude corresponding to ξ_n
$\sigma_n^* = \sigma_n 10^3/E$	dimensionless stress of mode number n
σ_m^*	min (σ_1^*, σ_2^*, σ_3^*)
σ_s^*	limit dimensionless stress for imperfect column (load carrying capacity)

1. INTRODUCTION

Thin-walled elements are widely used as structural components in many types of metal structures in which the interaction buckling of elastic columns may result in an imperfection-sensitive structure and is the principal cause of collapse of thin-walled structures.

The nonlinear theory of stability by Byskov and Hutchinson (1977) is based on asymptotic expansions of the postbuckling path and is capable of considering the interactive buckling. The expression for potential energy of the system expands in a series relative to the amplitudes of linear modes near the point of bifurcation; the latter generally corresponds to the minimum value of bifurcation loads.

For the first order approximation Koiter and van der Neut (1980) have proposed a technique in which the interaction of an overall mode with two local modes having the same wavelength has been considered. The fundamental mode is, henceforth, called "primary" and the nontrivial higher mode having the same wavelength as the "primary" is called "secondary".

In the energy expression for the first order approximation the coefficients of the cubic terms $\xi_1\xi_2^2, \xi_1\xi_3^2$ and $\xi_1\xi_2\xi_3$ (where the index is 1 for the global mode, 2 for the primary local buckling mode and 3 for the secondary local mode) are the key terms governing the interaction. In the case of disregarding the interaction between the overall mode and the primary local mode and the secondary local mode, the coefficient of the $\xi_1\xi_2\xi_3$ term in the energy expression vanishes. In the analysis of columns with doubly symmetric cross-sections the coefficients of the $\xi_1\xi_2^2$ and $\xi_1\xi_3^2$ terms vanish.

If the analysis is restricted to the first order approximation in solving the stability problem of the thin-walled structures the imperfection sensitivity can only be obtained. The structures where local buckling preceedes the global one, $\lambda_1 \gg \lambda_2$ ($\sigma_1^* \gg \sigma_2^*$) are widely used. It is well known that perfect structures with $\lambda_1 \gg \lambda_2$ can carry a load higher than that referring to the bifurcation value of local buckling. The behaviour of such structures cannot always be analysed in terms of the first nonlinear approximation where the limit load is always lower than the minimum value of bifurcational load in the linear analysis. But such an estimation enables us to avoid serious numerical problems occuring when the second order solution is considered for typical thin-walled structures.

A determination of the post-buckling equilibrium path requires the second order approximation to be taken into account. Therefore it is necessary to consider the second order of approximation, that is the fourth order components in the potential energy (coefficients of the terms $\xi_i^2\xi_j^2$). In general the stability analysis with regard to the second nonlinear approximation requires the solution of boundary value problems: for second order local, global and mixed modes (associated with the coefficients of the terms ξ_1^4, ξ_i^4 and $\xi_1^2\xi_i^2$ (where $i > 1$) in the energy expression, respectively). However, in the case when $\lambda_1 \gg \lambda_2$, the most significant are local second order modes. The second order global mode for Euler's model of column is zero and in the case of an exact solution it is of little importance. The local secondary buckling mode for the first order approximation is analogous to the mixed one. The omission of the second order mixed mode involving the coefficients of the $\xi_1^2\xi_i^2$ $(i > 1)$ and $\xi_i^2\xi_j^2$ $(i,j > 1$ and $i \neq j)$ terms in the expression for potential energy is possible owing to the fact that the coefficients of the cubic terms $\xi_1\xi_3^2$ and $\xi_1\xi_2\xi_3$ have already been included in the analysis. The admissibility of neglecting the mixed mode was shown by Koiter (1976), Sridharan and Peng (1989).

The analysis of the second nonlinear approximation considering only the local second order modes (i.e. primary and secondary modes) was undertaken by Kolakowski (1993a, 1993b). In these papers the transformation of local modes with the increase of load up to the load carrying capacity was considered.

The equations for local modes in the second approximation depend not only on respective first order local modes, but, regarding the orthogonality conditions, also on the considered three first order modes. Therefore, none of the local second order modes obtained with allowance for interactive buckling is identical with the mode obtained according to the theory of single-mode buckling (an uncoupled buckling), where the condition of orthogonality in relation to the global mode is not binding.

A complete range of behaviour of the thin-walled structures from global to local stability was described (Kolakowski (1993a, 1993b)). In these solutions obtained, the effects of interaction of certain modes having the same wavelength, the shear lag phenomenon and also the effect of cross-sectional distortions were included.

In the present paper a semi-analitycal method for the interactive buckling analysis of prismatic plate structures based on the linear analysis is proposed. Some examples are presented to compare the results of the present method with the currently available analysis results.

2. STRUCTURAL PROBLEM

The long thin-walled prismatic elastic beam-columns of length l, composed of plane, rectangular plate segments interconnected along longitudinal edges, simply supported at both ends were considered by Kolakowski (1993a, 1993b). A plate model was adopted for the beam-column. For the i-th wall precised geometrical relationships were assumed to take into account both out-of-plane and in-plane bending:

$$\begin{aligned}
\varepsilon_{ix} &= u_{i,x}+0.5(v_{i,x}^2+w_{i,x}^2),\\
\varepsilon_{iy} &= v_{i,y}+0.5(u_{i,y}^2+w_{i,y}^2),\\
2\varepsilon_{ixy} &= \gamma_{ixy} = u_{i,y}+v_{i,x}+w_{i,x}w_{i,y},\\
\chi_{ix} &= -w_{i,xx},\quad \chi_{iy} = -w_{i,yy},\quad \chi_{ixy} = -w_{i,xy}.
\end{aligned} \tag{1}$$

The differential equilibrium equations resulting from the virtual work principle and corresponding to expressions (1) for the i-th wall could be written as follows:

$$\begin{aligned}
N_{ix,x}+N_{ixy,y}+(N_{iy}u_{i,y})_{,y} &= 0,\\
N_{iy,y}+N_{ixy,x}+(N_{ix}v_{i,x})_{,x} &= 0,\\
D_i\nabla\nabla w_i-(N_{ix}w_{i,x})_{,x}-(N_{iy}w_{i,y})_{,y}-(N_{ixy}w_{i,x})_{,y}-(N_{ixy}w_{i,y})_{,x} &= 0.
\end{aligned} \tag{2}$$

The solution of these equations for each plate should satisfy kinematic and static conditions at the junctions of adjacent plates and boundary conditions at the ends $x = 0$ and $x = l$.

The influence of modification of boundary conditions at both ends of the structures regarding interactive buckling were studied by Kolakowski (1993b). In this analysis two different boundary conditions referring to the simply support at its both ends were assumed:

— Analysis I (see also Kolakowski (1993a))

$$\begin{aligned}
N_{ix}(x_i = 0, y_i) &= N_{ix}(x_i = l, y_i) = \lambda N_{ix}^o,\\
v_i(x_i = 0, y_i) &= v_i(x_i = l, y_i) = 0,\\
w_i(x_i = 0, y_i) &= w_i(x_i = l, y_i) = 0,\\
w_{i,xx}(x_i = 0, y_i) &= w_{i,xx}(x_i = l, y_i) = 0.
\end{aligned} \tag{3a}$$

— Analysis II

$$\begin{aligned}
\frac{1}{b_i}\int N_{ix}(x_i = 0, y_i)\,\mathrm{d}y_i &= \frac{1}{b_i}\int N_{ix}(x_i = l, y_i)\,\mathrm{d}y_i = \lambda N_{ix}^o,\\
v_i(x_i = 0, y_i) &= v_i(x_i = l, y_i) = 0,\\
w_i(x_i = 0, y_i) &= w_i(x_i = l, y_i) = 0,\\
w_{i,xx}(x_i = 0, y_i) &= w_{i,xx}(x_i = l, y_i) = 0.
\end{aligned} \tag{3b}$$

The non-linear problem was solved by the asymptotic Byskov and Huchinson method (1977). Displacement $\bar{U}$ and force $\bar{N}$ fields were expanded in power series in the buckling mode amplitudes, ξ_n (the amplitude of the n-th buckling mode was divided by the thickness of the first component plate, h_1):

$$\begin{aligned}
\bar{U} &= \lambda\bar{U}_i^{(0)}+\xi_n\bar{U}_i^{(n)}+\xi_n^2\bar{U}_i^{(nn)}+\ldots,\\
\bar{N} &= \lambda\bar{N}_i^{(0)}+\xi_n\bar{N}_i^{(n)}+\xi_n^2\bar{N}_i^{(nn)}+\ldots,
\end{aligned} \tag{4}$$

where $\bar{U}_i^{(0)}, \bar{N}_i^{(0)}$ are the pre-buckling fields, $\bar{U}_i^{(n)}, \bar{N}_i^{(n)}$ the buckling modes and $\bar{U}_i^{(nn)}, \bar{N}_i^{(nn)}$ the post-buckling fields. The range of indicies was [1, N] where N was the number of interacting modes.

By substituting the expansion (4) into the equations of equilibrium (2), junction conditions and boundary conditions, boundary problems of zero, first and second order could be obtained.

The zero approximation described the pre-buckling state while the first approximation, that was the linear problem of stability and was able to determine the critical value and the buckling mode, was reduced to a system of ordinary homogeneous differential equations. This system with appropriate junction conditions for adjacent plates was solved by the transition matrices method using numerical integration of the equilibrium equations in a transverse direction in order to obtain a relation between state vectors on two longitudinal edges.

The second order boundary problem could be reduced to a linear system of non-homogeneous equations whose right-hand sides depended on displacement field and first order forces. Solution of this system was sought in the form of trigonometric series, the functions defining second order displacement having been determined in the same way as the first order field (Kolakowski 1993b).

At the point where load parameter λ reaches its maximum value λ_s for the imperfect structure (secondary bifurcation or limit points) the Jacobian of the non-linear system of equations (Byskov and Hutchinson (1977)):

$$a_J\left(1-\frac{\lambda}{\lambda_J}\right)\xi_J+a_{ijJ}\xi_i\xi_j+b_{JJJJ}\xi_J^3+\cdots = a_J\frac{\lambda}{\lambda_J}\bar{\xi}_J \quad \text{at } J = 1,2\ldots N \tag{5}$$

is equal to zero.

The corresponding expression for the total potential energy of the structures have the following form:

$$\Pi = -a_o\lambda^2/2+\sum_J^N a_J\xi_J^2(1-\lambda/\lambda_J)/2+a_{ijk}\xi_i\xi_j\xi_k/3+\sum_J^N b_{JJJJ}\xi_J^4/4-\sum_J^N a_J\xi_J\bar{\xi}_J\lambda/\lambda_J, \tag{6}$$

where λ is the load parameter, λ_J is the critical value of λ and $\Pi_o = a_o\lambda^2/2$ is the energy of pre-buckling state. The coefficient $b_{1111} = 0$ was assumed.

Expressions for a_{ijJ} and b_{ijkJ} are given in the Appendix. The formulae for the post-buckling coefficients a_{ijJ} depended only on the buckling modes whereas coefficients b_{JJJJ} also depend on the second order field.

The various boundary conditions (3a) and (3b) caused only different values of the loading carrying capacity σ_s^* for the second order approximation.

The character of the second-order field determining the postbuckling behaviour of uncoupled local buckling modes varies, subject to the mode of first-order buckling in question. In the presented semi-analitycal method the buckling displacement components in the first order approximation are neglected as distinct from papers by Kolakowski (1993a, b) and Kolakowski and Teter (1995). In the second order approximation the postbuckling membrane energy associated with the stress components $N_{iy}^{(nn)}$ and $N_{ixy}^{(nn)}$ is neglected as by Koiter and Pignataro (1976). The second order stress is the sum of two components $\sigma^{(nn)} = \sigma_p^{(nn)}+\sigma_h^{(nn)}$, where $\sigma_p^{(nn)}$ refers to a particular solution of the compatibility equation and $\sigma_h^{(nn)}$ represents the solution of the homogeneous equation (boundary effect) which is neglected in the present method. In this case relation defines the post-buckling coefficient $\bar{b}_{KKKK}$ in the following form:

$$\bar{b}_{KKKK} \approx \sigma^{(nn)}\cdot 1_2(U^{(n)}) = N_{ix}^{(nn)}\cdot 1_2(U^{(n)}) \approx \sum_i \frac{Eh_i l}{8}\left\{\frac{m\pi}{l}\right\}^4 \int_0^{b_i} [W_i^{(K)}(y_i)]^4\, dy_i \tag{7}$$

for the general K-th local buckling mode described by:

$$w_i^{(K)}(x, y_i) = W_i^{(K)}(y_i)\sin\frac{m\pi x}{l}, \tag{8}$$

where m is the number of axial half-waves of the local buckling mode and $W_i^{(K)}(y_i)$ is the function which specifies the shape of the K-th uncoupled local buckles across the i-th wall of the beam-columns for the first order displacement (Kolakowski and Teter (1995)).

Overall flexural stiffness is determined by the generalized column edge displacement corresponding the K-th uncoupled local buckling i.e. single-mode local buckling (where index K is 2 for the primary local buckling mode, 3 for the secondary local mode) which equals $\Delta_K = \partial\Pi/\partial\lambda_K$. Equation (6) gives:

$$\Delta_K = -a_o\lambda/\lambda_K^2 - a_K\xi_K^2/(2\lambda_K) - a_K\xi_K\bar{\xi}_K/\lambda_K. \tag{9}$$

The generalized displacement up to the uncoupled local buckling for the unbending pre-buckling state is:

$$\Delta_{Ko} = -a_o\lambda/\lambda_K^2. \tag{10}$$

The generalized displacements, Δ_{Ko}/Δ_K defines a coefficient of the reduced flexural rigidity, $\eta_K = I_*/I$, where I_* is the effective moment of the inertia of the cross-section:

$$\eta_K = \left[1 + \frac{a_K\xi_K\lambda_K}{a_o\lambda}(0.5\xi_K + \bar{\xi}_K)\right]^{-1}. \tag{11}$$

In a special case of the ideal beam-column (that means $\bar{\xi}_K = 0$) and of the symmtrical characteristic relative to the deflections for the local mode ($a_{KKK} = 0$):

$$\bar{\eta}_K = \lim_{\lambda\to\infty}\eta_K = \left[1 + \frac{a_K^2}{2a_o b_{KKKK}}\right]^{-1} \approx \left[1 - \frac{a_K^2}{2a_o\bar{b}_{KKKK}}\right]. \tag{12}$$

For a single compressed plate the eqn (12) reduces to the known form by Koiter and Pignataro (1976) (see also Reis (1987)) and this value of $\bar{\eta}_K$ is a "lower bound" with respect to the exact value:

$$\bar{\eta}_K = 1 - (\bar{W}_K^2)^2/\bar{W}_K^4, \tag{13}$$

where $\bar{W}_K^2$ and $\bar{W}_K^4$ are the average value of $[W_i^{(K)}(y)]^2$ and $[W_i^{(K)}(y)]^4$, respectively:

$$\bar{W}_K^2 = \frac{1}{b_i}\int_0^{b_i}[W_i^{(K)}(y_i)]^2\,dy_i,$$

$$\bar{W}_K^4 = \frac{1}{b_i}\int_0^{b_i}[W_i^{(K)}(y_i)]^4\,dy_i. \tag{14}$$

In this case a constant value of $\bar{\eta}_K$ is obtained.

In the presented semi-analitycal method it is postulated to determine approximately the value of b_{KKKK} for the K-th uncoupled local field of the second order resulting from eqn (12) as follows:

$$b_{KKKK} = \bar{b}_{KKKK} - a_K^2/(2a_o). \tag{15}$$

A semi-analytical method enabling an approximate analysis of interactive buckling in terms of the second order non-linear approximation, regarding only linear analysis, consists in the determination of the post-buckling coefficient b_{KKKK} from (15). This coefficient should be substituted into the equations describing the post-buckling equilibrium path (5).

Such an estimation allows to avoid a number of theoretical and numerical problems connected with the second order of approximation, and does not require the orthogonalization of the second order fields in relation to the first order ones. This method, however, does not take into account the transformation of local buckling modes which accompanies the increase in the load parameter λ and may be applied for structures subjected to compression and bending.

For the second order of approximation Kolakowski (1993a, 1993b) obtained the non-linear coefficients b_{3333} being in some cases lower than zero which resulted in a reduction of the load carrying capacity σ_s^* as compared with the case when $b_{3333} = 0$. On the other hand, if the coefficient b_{3333} is higher than zero, σ_s^* increases in relation to $b_{3333} = 0$. An estimation of the value of the coefficient b_{KKKK} on the basis of (15) does not enable us to consider the influence of the imperfections and of the post-buckling coefficient a_{KKK} (for a more detailed analysis see Manevich and Kolakowski (1995)). The coefficient b_{KKKK} obtained from (15) does not depend on the value of load, λ.

The above considerations suggest that exclusively in terms of the second order approximation only the basic buckling mode (that is $b_{2222} \neq 0$, $b_{3333} = 0$) should be taken into account; this allows us to obtain the lower estimation of load carrying capacity σ_s^* for the assumed global and local imperfections.

The analysis carried out for single-mode buckling (Manevich and Kolakowski (1995)) suggests that it is necessary to consider the influence of the imperfections upon the reduced flexural rigidity η_K.

In case of single-mode local buckling it is also possible to consider indirectly the effect of the imperfections on the global post-buckling flexural rigidity. The value of b_{2222} obtained from relation (15) should be substituted to eqn (5), restricted to the case of an uncoupled buckling, and to eqn (11), in order to find a 'corrected' value of reduced flexural rigidity η_2 for a given load λ/λ_2 and assumed local imperfection ξ_2 (like in Manevich and Kolakowski (1995)).

3. ANALYSIS OF RESULTS

In the numerical analysis a comparison has been presented between data calculated according to the semi-analytical method outlined here and those obtained for the second order approximation by Kolakowski (1993a, 1993b).

3.1. *Closed column*

Geometrical dimensions of the cross-section of a thin-walled column subject to compression are presented in Fig. 1.

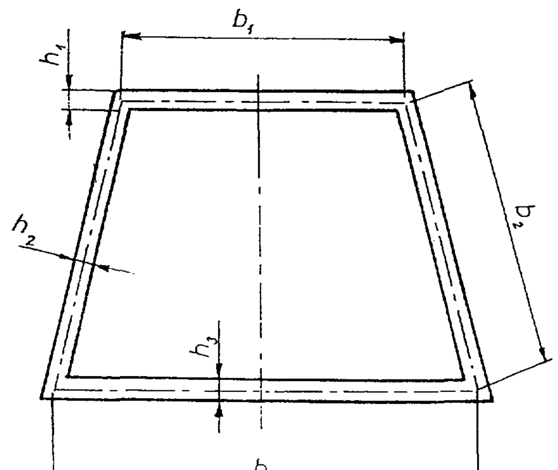

Fig. 1. Closed column geometry.

Table 1. Load-carrying capacity for the square column of the following cross-section dimensions: $b_1/b_2 = b_3/b_2 = 1.0$, $h_1/h_2 = h_3/h_2 = 1.0$, $l/b_2 = 67.39$, $b_2/h_2 = 100.0$ at imperfections $|\bar{\xi}_1| = 1.0$, $|\bar{\xi}_2| = 0.2$, $\bar{\xi}_3 = 0.0$

	Linear analysis			Nonlinear analysis		
m	σ_1^*	σ_2^*	σ_3^*	σ_s^*/σ_m^* Analysis I	σ_s^*/σ_m^* Analysis II	σ_s^*/σ_m^* Present method
67	0.3614	0.3614	0.5199	0.7433	0.7426	0.7419
80	0.3614	0.3722	0.4904	0.7258	0.7253	0.7244

Table 1 presents non-dimensional bifurcational stresses (linear problem) for a square column which geometrical dimensions are (Kolakowski (1993b)):

$$b_1/b_2 = 1.0, \quad b_3/b_2 = 1.0, \quad h_1/h_2 = 1.0,$$
$$h_3/h_2 = 1.0, \quad l/b_2 = 67.39, \quad b_2/h_2 = 100.0, \quad \nu = 0.3,$$

as well as the ratio of limit stress to the minimum value of bifurcational stress in the second order σ_s^*/σ_m^* for Analysis I, Analysis II and the presented method, considering the interaction of three buckling modes (index n is 1 for the global mode, 2 for the primary local mode and 3 for the secondary local mode having the same number of half-waves as the primary one), the assumed imperfections being $|\bar{\xi}_1| = 1.0$, $|\bar{\xi}_2| = 0.2$, $\bar{\xi}_3 = 0.0$. At $m = 80$ the load carrying capacity σ_s^* reaches its minimum value for the assumed level of imperfections.

Table 2 shows similar calculation results obtained for a trapezoid-section column (Kolakowski (1993b)):

$$b_1/b_2 = 0.5237, \quad b_3/b_2 = 1.0474, \quad h_1/h_2 = 0.4651,$$
$$h_3/h_2 = 1.5814, \quad l/b_2 = 46.095, \quad b_2/h_2 = 88.8, \quad \nu = 0.3,$$

with the identical imperfection values.

It should be noted that geometrical dimensions of the cross-section were selected so that the moments of the inertia of the section relative to central axes of inertia could be identical.

In the case where $m = 69$ a minimum value of the limit load is reached analogous to the case of two fold axis of symmetry ($m = 80$, Table 1). In both cases they are obtained regarding the influence of imperfections and the number of m.

A very good conformity is obtained between the results of the method presented and those of Analysis I and Analysis II (maximum difference is about 5%).

As the values of the stress σ_1^* and σ_2^* are nearly simultaneous (Tables 1 and 2) this type of behaviour could already be analysed in the terms of the first order approximation.

A comparison between the results obtained in this paper and those by Kolakowski (1993a) for Analysis I and Sridharan and Ali (1986) for columns with a square section and

Table 2. Load-carrying capacity for the trapezoidal column of the following cross-section dimensions: $b_1/b_2 = 0.5237$, $b_3/b_2 = 1.0474$, $h_1/h_2 = 0.4651$ $h_3/h_2 = 1.5814$, $l/b_2 = 46.095$, $b_2/h_2 = 88.8$ at imperfections $|\bar{\xi}_1| = 1.0$, $|\bar{\xi}_2| = 0.2$, $\bar{\xi}_3 = 0.0$

	Linear analysis			Nonlinear analysis		
m	σ_1^*	σ_2^*	σ_3^*	σ_s^*/σ_m^* Analysis I	σ_s^*/σ_m^* Analysis II	σ_s^*/σ_m^* Present method
54	0.5678	0.5659	1.1929	0.8083	0.8263	0.8036
69	0.5678	0.5983	0.9578	0.7625	0.7756	0.7324

identical wall thickness is shown in Fig. 2. In this case the influence of numbers of half-waves, m, for assumed imperfections on the load carrying capacity is neglected. The ratio of the load carrying capacity to the primary local stress (σ_s^*/σ_2^*) is plotted against (σ_1^*/σ_2^*) in Fig. 2a, illustrating the effect of local imperfections, and in Fig. 2b the effect of overall imperfections for a fixed value of local imperfection, $\bar{\zeta}_2 = 0.025$, is shown. Data obtained with the use of this method are practically identical with those presented in Kolakowski (1993a). As a comparison, Fig. 2 shows calculation results obtained for the "lower bound" modelling of local buckling in Koiter's approach (Koiter and van der Neut (1980) (example by Sridharan and Ali (1986)). In this case it is seen that there is a good agreement for values of (σ_1^*/σ_2^*) in close proximity to 1.0 and for smaller magnitudes of imperfections. Discrepancies become larger with increasing (σ_1^*/σ_2^*) and with increasing global imperfection values.

In the present paper, "exact" continuity conditions are adopted for the longitudinal edges, the limit load capacity being, therefore, lower than that by Koiter and Neut (1980), where only simplified conditions were assumed. The Koiter and Neut approach is based on a mechanical model and asymptotic analysis.

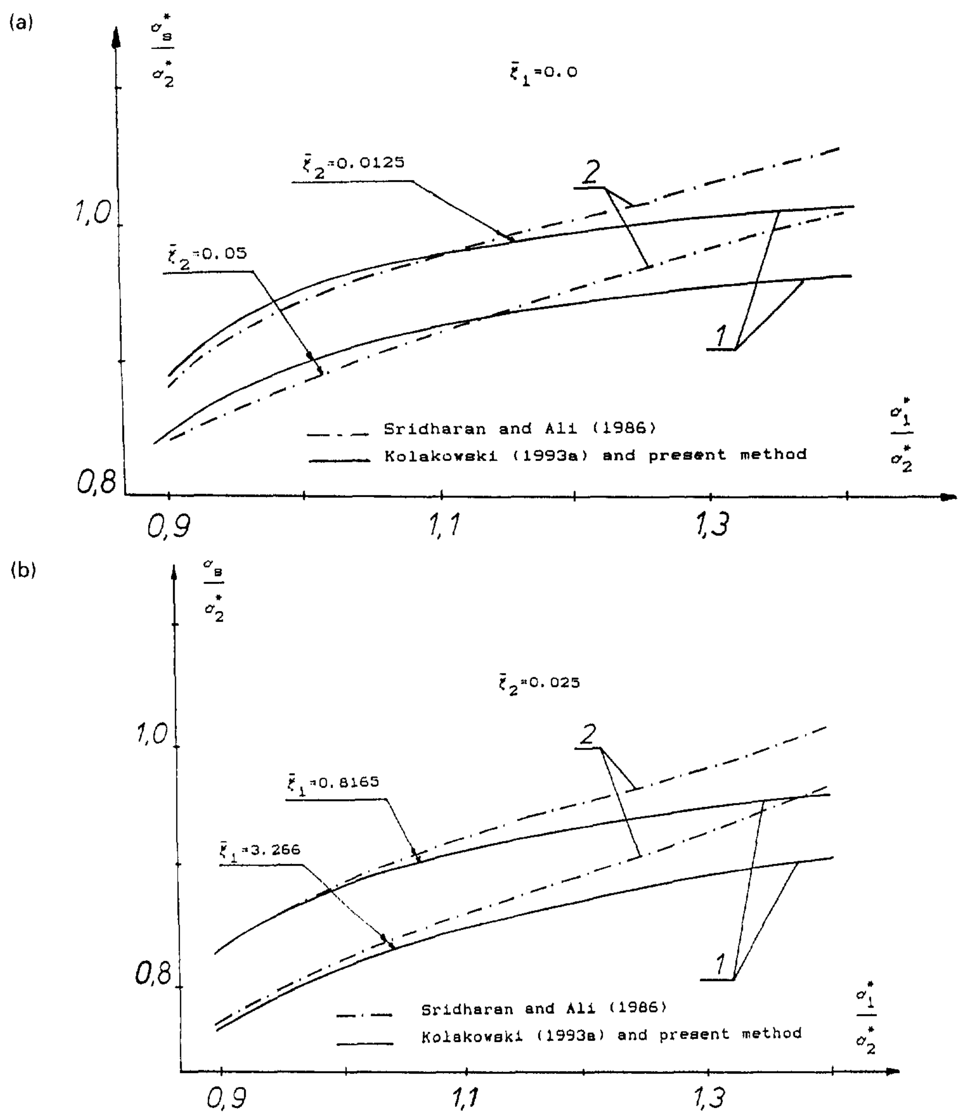

Fig. 2. Relationship between σ_s^*/σ_2^* and σ_1^*/σ_2^* for the square column. Curves: 1-Kolakowski (1993a), 2-Sridharan and Ali (1986).

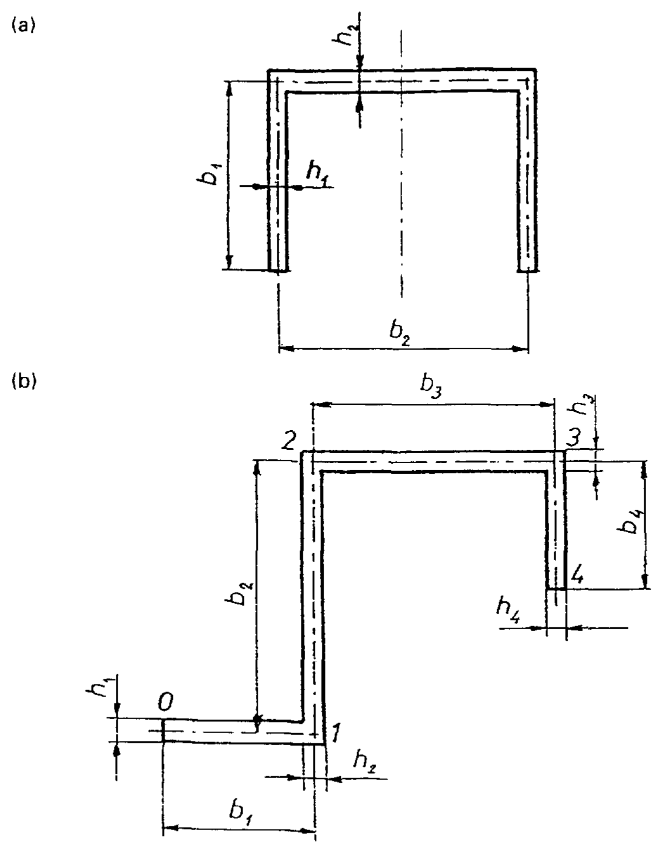

Fig. 3. Types of open cross-sections considered.

3.2. *Open section columns*

Geometrical dimensions of columns under discussion are shown in Fig. 3.

In thin-walled columns analysed by Ali and Sridharan (1988) and Kolakowski (1993b), their dimensions being as follows (Fig. 3a):

$$b_1/b_2 = 0.5, \quad h_1/h_2 = 1.0, \quad l/b_2 = 78, \quad b_2/h_2 = 50.0, \quad \nu = 0.3,$$

the ratio of the global stress value corresponding to the flexural-torsional buckling (the primary global mode) to the stress value of the primary local mode is 2.385, while the ratio of the stress of the global Euler mode (the secondary global mode) to the stress of the primary local mode is 3.545.

Table 3 presents non-dimensional stresses σ_n^* at the number of half-waves being $m = 6$ and $m = 8$ and the load carrying capacity σ_s^* for Analysis I, Analysis II and the presented

Table 3. Load-carrying capacity for open column of the following cross-section dimensions: $b_1/b_2 = 0.5$, $h_1/h_2 = 1.0$, $l/b_2 = 7.8$, $b_2/h_2 = 50.0$ at imperfections $|\xi_1| = 1.0$, $|\xi_2| = 0.2$, $\xi_3 = 0.0$

	Linear analysis			Nonlinear analysis		
m	σ_1^*	σ_2^*	σ_3^*	σ_s^*/σ_m^* Analysis I	σ_s^*/σ_m^* Analysis II	σ_s^*/σ_m^* Present method
6	2.508(a)	1.051(s)	1.440(a)	0.856	1.756	0.935
6	3.696(s)	1.051(s)	1.440(a)	0.746	0.994	0.920
6	3.696(s)	1.051(s)	3.308(s)	0.731	0.919	0.826
8	2.508(a)	1.131(s)	1.374(a)	0.860	1.250	0.924
8	3.696(s)	1.131(s)	1.374(a)	0.767	0.982	0.926
8	3.696(s)	1.131(s)	2.513(s)	0.741	0.898	0.826

method at the imperfections $|\bar{\xi}_1| = 1.0$, $|\bar{\xi}_2| = 0.2$, $\bar{\xi}_3 = 0.0$ and for some possible combinations of buckling modes. In brackets, the following code has been used in Table 3 in order to identify the support conditions on the axis of the symmetry of the cross-section: a—antisymmetry, s—symmetry, respectively, for the n-th buckling mode.

The case $m = 8$ refers to the minimum value of the secondary local buckling mode.

The limit carrying capacity σ_s^* is determined by an interaction of the more dangerous secondary global (Euler's) buckling mode with two symmetric local modes.

Table 4 contains calculation results analogous to those from Table 3, with identical imperfection values, but for a column of the following dimensions (Kolakowski (1993b):

$$b_1/b_2 = 0.3077,\quad h_1/h_2 = 0.8,\quad l/b_2 = 5.0,\quad b_2/h_2 = 52.0,\quad \nu = 0.3.$$

The primary global buckling mode refers to Euler's one, while the secondary global mode corresponds to the flexural-torsional one. Also in this case a good separation of global modes from the primary local mode is present.

Similarly to the results presented in Table 3, lower values of the limit load are obtained in case of the interaction of the global Euler mode with two local symmetrical modes than in case of the interaction of the global flexural-torsional one with local ones.

In case of open-section columns the method presented has been found to be in a good agreement with the author's earlier papers. The load carrying capacity determined by approximate method lie between the results obtained in Analysis I and Analysis II.

Table 5 contains the results obtained for an open-section beam under eccentric compression (Fig. 3b), its geometrical dimensions being as follows (Kolakowski (1993a)):

$$b_2/b_1 = b_3/b_1 = 3.0,\quad b_4/b_1 = 1.0,\quad l/b_1 = 20.0,$$
$$h_2/h_1 = h_3/h_1 = h_4/h_1 = 1.0,\quad b_2/h_2 = 100.0,\quad \nu = 0.3.$$

The following data are listed in the table: ratio of stress σ_i^* at point i to maximum stress σ_{max}^* applied to the beam; values of nondimensional stresses σ_n^*; number of half-waves, m, corresponding to the local buckling mode and the ratio of σ_s^*/σ_2^* for Analysis I and the present approach, assuming the following imperfections $|\bar{\xi}_1| = 1.0$, $|\bar{\xi}_2| = 0.2$ and $\bar{\xi}_3 = 0.0$. In this case the influence of numbers of half-waves m for assumed imperfections on the load carrying capacity is neglected (as by Kolakowski (1993a)).

A comparison of results concerns only Analysis I and the method described here, therefore it is not possible to make a comparison with results for Analysis II.

It is easy to notice that taking into account the second order approximation where a good mode separation is present, $\sigma_1^*/\sigma_2^* > 3.9$, at the assumed imperfection level, the limit load does not exceed for Analysis I $\sigma_s^*/\sigma_2^* < 1.6$ and the present approach $\sigma_s^*/\sigma_2^* < 1.85$.

The presented semi-analytical method for the analysis of interactive buckling in the second order approximation, in terms of linear analysis, leads to a correct estimation of load carrying capacity σ_s^* in cases under consideration (see also Manevich and Kolakowski (1995)).

Table 4. Load carrying capacity for open column of the following cross-section dimensions: $b_1/b_2 = 0.3077$, $h_1/h_2 = 0.8$, $l/b_2 = 5.0$, $b_2/h_2 = 52.0$ at imperfections $|\bar{\xi}_1| = 1.0$, $|\bar{\xi}_2| = 0.2$, $\bar{\xi}_3 = 0.0$

	Linear analysis			Nonlinear analysis		
m	σ_1^*	σ_2^*	σ_3^*	σ_s^*/σ_m^* Analysis I	σ_s^*/σ_m^* Analysis II	σ_s^*/σ_m^* Present method
5	2.474(s)	1.289(s)	2.737(a)	1.738	1.750	1.730
5	3.541(a)	1.289(s)	2.737(a)	1.408	1.747	1.413
5	2.474(s)	1.289(s)	3.828(s)	1.157	1.185	1.150
8	2.474(s)	1.612(s)	2.267(a)	1.502	1.878	1.507
8	3.541(a)	1.612(s)	2.267(a)	1.332	1.515	1.377
8	2.474(s)	1.612(s)	2.726(s)	1.089	1.110	1.096

Table 5. Load carrying capacity for open beam of following cross-section dimensions $b_2/b_1 = b_3/b_1 = 3.0$, $b_4/b_1 = 1.0$, $h_2/h_1 = h_3/h_1 = h_4/h_1 = 1.0$, $b_2/h_2 = 100.0$, $l/b_1 = 20.0$ at imperfections $|\bar{\xi}_1| = 1.0$, $|\bar{\xi}_2| = 0.2$, $\bar{\xi}_3 = 0.0$

Loading eccentricity					Linear analysis					Nonlinear analysis	
		$\sigma_i^*/\sigma_{max}^*$								σ_s^*/σ_2^*	σ_s^*/σ_2^*
0	1	2	3	4	σ_1^*	σ_2^*	σ_3^*	m	σ_1^*/σ_2^*	Analysis I	Present method
1.0	1.0	1.0	1.0	1.0	1.816	0.374	0.981	6	4.85	1.460	1.155
0.198	0.5	−0.115	0.794	1.0	3.238	0.699	1.665	7	4.63	0.761	1.349
0.189	−0.054	0.596	1.0	0.784	2.603	0.519	1.100	7	5.02	1.493	1.118
0.369	0.722	0.455	0.606	1.0	3.909	0.791	4.160	8	4.94	0.736	1.451
0.026	−0.205	1.0	0.305	−0.097	5.046	0.686	1.310	7	7.35	1.510	1.847
0.133	−0.241	1.0	−0.122	−0.536	8.479	0.911	1.446	8	9.31	1.563	1.540
0.619	0.251	1.0	−0.106	−0.356	4.253	0.665	1.670	7	6.39	1.527	1.499
1.0	0.882	0.343	−0.012	0.168	2.664	0.583	1.134	7	4.57	0.811	1.034
0.772	1.0	−0.204	0.483	0.885	2.852	0.729	1.637	7	3.91	0.756	0.967

The presented method enables a full analysis of the interactive buckling in beam-columns to be carried out, making allowance for global pre-buckling bending (see also Roorda (1988)).

4. CONCLUSIONS

The present paper deals with a semi-analytical method for the analysis of the post-buckling behaviour of open- and closed-section thin-walled structures subject to compression and bending.

In case the global stress exceeds the local one, it is possible to attain the load carrying capacity higher than the minimum local stress value for the proposed semi-analytical method.

The numerical analysis shows that values of coefficients b_{KKKK} are nearly equal or smaller than their values obtained for Analysis II (Kolakowski (1993b)).

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APPENDIX

The coefficients in the equilibrium equations (5) are given by following expressions (see Byskov and Hutchinson (1977) for more detailed analysis):

$$a_o = \sigma^{(o)} \cdot \varepsilon^{(o)}$$

$$a_J = -\lambda_J \sigma^{(o)} \cdot l_2(U^{(J)})$$

$$a_{ijJ} = 0.5\sigma^{(J)} \cdot l_{11}(U^{(i)}, U^{(j)}) + \sigma^{(i)} \cdot l_{11}(U^{(j)}, U^{(J)})$$

$$b_{ijkJ} = \sigma^{(ij)} \cdot l_{11}(U^{(k)}, U^{(J)}) + 2\sigma^{(i)} \cdot l_{11}(U^{(J)}, U^{(jk)})$$